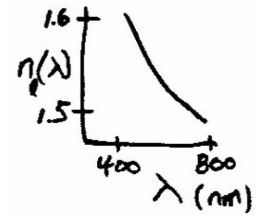
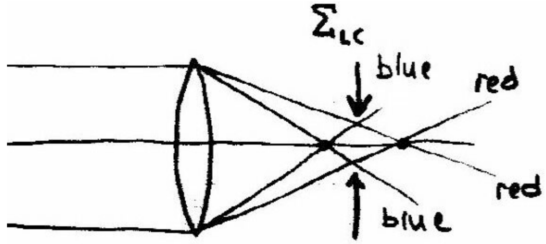
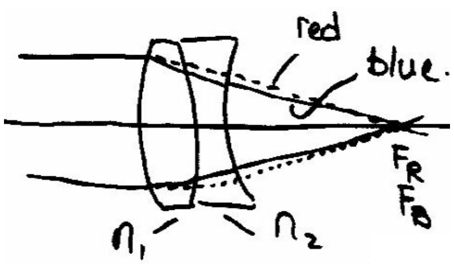
Chromatic Abberations

$$\frac{1}{f} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$





use achromatic doublet to minimize aberration:



write

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

and examine $f(\lambda)$

will have $F_y \neq F_R$ or F_B

lens 1: $\rho_1 \equiv \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, lens 2: $\rho_2 \equiv \left(\frac{1}{R_2} - \frac{1}{R_3}\right)$

then $\frac{1}{f} = (n_1 - 1)\rho_1 + (n_2 - 1)\rho_2$

specify indices: red: n_{1R}, n_{2R}

blue: n_{1B}, n_{2B}

demand $\frac{1}{f_R} = \frac{1}{f_B}$

$$\Rightarrow (n_{1R} - 1)\rho_1 + (n_{2R} - 1)\rho_2 = (n_{1B} - 1)\rho_1 + (n_{2B} - 1)\rho_2$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = -\frac{n_{2B} - n_{2R}}{n_{1B} - n_{1R}} \quad (*)$$

note: $\rho_1 > 0$, $\rho_2 < 0$ ✓

can determine $\frac{\rho_1}{\rho_2}$ given the n's.

Specify focal length of compound lens with yellow light;

$$\frac{1}{f_{1y}} = (n_{1y} - 1)\rho_1, \quad \frac{1}{f_{2y}} = (n_{2y} - 1)\rho_2$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{(n_{2y} - 1) f_{2y}}{(n_{1y} - 1) f_{1y}} \quad \text{equate with } (*)$$

$$\Rightarrow \frac{f_{2y}}{f_{1y}} = -\underbrace{\frac{(n_{2B} - n_{2R})}{(n_{2y} - 1)} \bigg/ \frac{(n_{1B} - n_{1R})}{(n_{1y} - 1)}}_{\text{dispersive powers (tabulated)}}$$

In practise, designate specific spectral lines:

TABLE 6.1 Several Strong Fraunhofer Lines

Designation	Wavelength (Å)*	Source
C	6562.816 Red	H
D_1	5895.923 Yellow	Na
D	Center of doublet 5892.9	Na
D_2	5889.953 Yellow	Na
D_3 or d	5875.618 Yellow	He
b_1	5183.618 Green	Mg
b_2	5172.699 Green	Mg
c	4957.609 Green	Fe
F	4861.327 Blue	H
f	4340.465 Violet	H
g	4226.728 Violet	Ca
K	3933.666 Violet	Ca

*1 Å = 0.1 nm.

$V_D \equiv$ Abbe number $= (\text{dispersive power})^{-1}$

$$\Rightarrow \frac{f_{2y}}{f_{1y}} = -\frac{V_1}{V_2} \quad \text{or} \quad \underline{f_{1y}V_1 + f_{2y}V_2 = 0} \quad (*)$$

example:

518/596 crown and 617/366 flint

$1000(n_D - 1)$ $\quad$ $10V$

want $f = 15$ cm, $\frac{1}{f} = \frac{1}{f_{1y}} + \frac{1}{f_{2y}}$

$$\text{with } (*) \Rightarrow f_{1y} = 15\left(1 - \frac{V_2}{V_1}\right) = 15\left(1 - \frac{366}{596}\right) = 5.79 \text{ cm}$$

$$f_{2y} = 15\left(1 - \frac{V_1}{V_2}\right) = 15\left(1 - \frac{596}{366}\right) = -9.42 \text{ cm}$$

say $R_2 = -R_1$, $\frac{1}{5.79} = .518\,\frac{2}{R_1} \Rightarrow R_1 = 6.00 \text{ cm} = -R_2$

and $\frac{1}{-9.42} = .617\left(-\frac{1}{6.00} - \frac{1}{R_3}\right) \Rightarrow R_3 = 185.6$ cm.

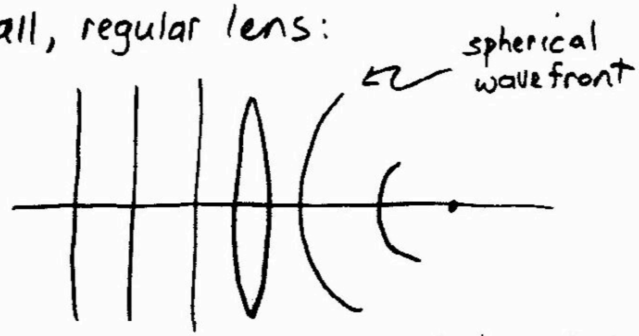
TABLE 6.2 Optical Glass

Type Number	Name	n_D	V_D
511:635	Borosilicate crown—BSC-1	1.5110	63.5
517:645	Borosilicate crown—BSC-2	1.5170	64.5
513:605	Crown—C	1.5125	60.5
518:596	Crown	1.5180	59.6
523:586	Crown—C-1	1.5230	58.6
529:516	Crown flint—CF-1	1.5286	51.6
541:599	Light barium crown—LBC-1	1.5411	59.9
573:574	Barium crown—LBC-2	1.5725	57.4
574:577	Barium crown	1.5744	57.7
611:588	Dense barium crown—DBC-1	1.6110	58.8
617:550	Dense barium crown—DBC-2	1.6170	55.0
611:572	Dense barium crown—DBC-3	1.6109	57.2
562:510	Light barium flint—LBF-2	1.5616	51.0
588:534	Light barium flint—LBF-1	1.5880	53.4
584:460	Barium flint—BF-1	1.5838	46.0
605:436	Barium flint—BF-2	1.6053	43.6
559:452	Extra light flint—ELF-1	1.5585	45.2
573:425	Light flint—LF-1	1.5725	42.5
580:410	Light flint—LF-2	1.5795	41.0
605:380	Dense flint—DF-1	1.6050	38.0
617:366	Dense flint—DF-2	1.6170	36.6
621:362	Dense flint—DF-3	1.6210	36.2
649:338	Extra dense flint—EDF-1	1.6490	33.8
666:324	Extra dense flint—EDF-5	1.6660	32.4
673:322	Extra dense flint—EDF-2	1.6725	32.2
689:309	Extra dense flint—EDF	1.6890	30.9
720:293	Extra dense flint—EDF-3	1.7200	29.3

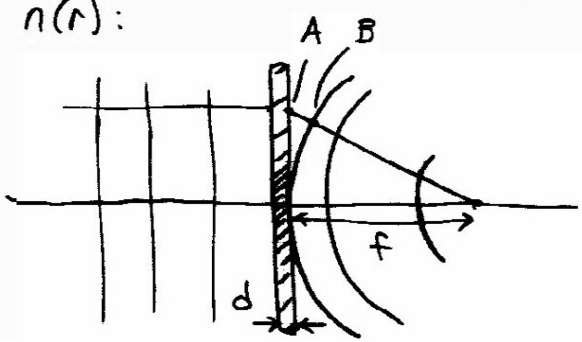
From T. Calvert, "Optical Components," *Electromechanical Design*, May 1971. Type number is given by (n_D − 1):(10 V_D), where n_D is rounded off to three decimal places. For more data, see Smith, *Modern Optical Engineering*, Fig. 7.5.

Graded Index of Refraction (GRIN) Lenses:

recall, regular lens:



could accomplish same thing with varying $n(r)$:



equate optical path lengths for any r:

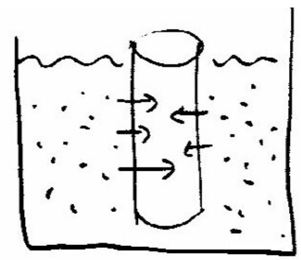
$$(OPL)_r + \overline{AB} = (OPL)_0$$

$$n(r)d + \overline{AB} = n_0 d$$

$$\sqrt{r^2+f^2} - f \approx f\left(1+\frac{r^2}{f^2}\right)^{1/2} - f \approx \frac{1}{2}\frac{r^2}{f}$$

thus, $\underline{\underline{n = n_0 - \frac{r^2}{2fd}}}$

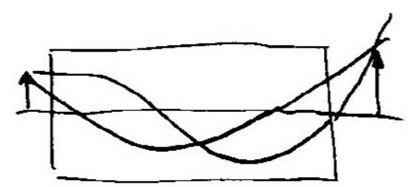
obtain this profile by diffusion of impurities into a glass fiber:



high T

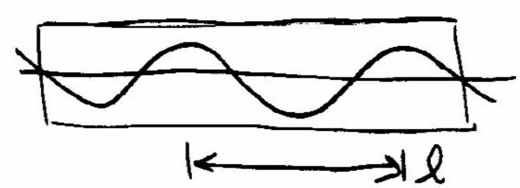
typically $\Delta n \simeq 0.1$ and diameter is limited

- can also produce thick elements with performance unlike lenses:



eg. copying machine

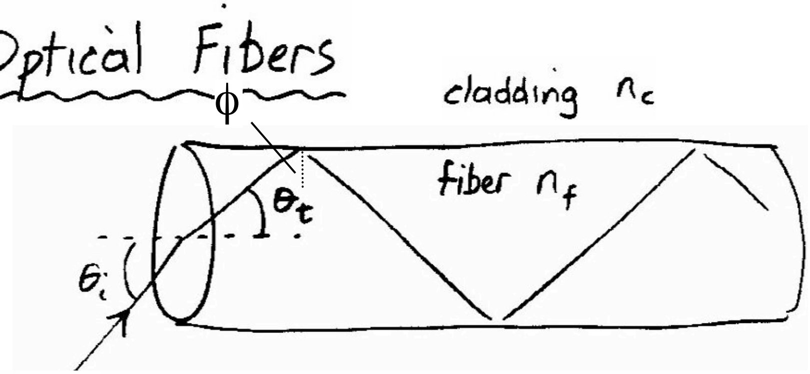
or, GRIN fiber,



with $n = n_0\left(1 - \frac{ar^2}{2}\right)$

then $\ell = 2\pi/\sqrt{a}$

Optical Fibers



- for $n_c < n_f$ have total internal reflection for $\theta_i < \theta_{max}$ such that $\frac{\pi}{2} - \theta_t \equiv \phi \leq \phi_c$
- light rays can propagate with little loss, and high freq. allow high info-carrying capacity

eg. TV station - 6 Mz bandwith $\equiv \Delta f$

radio waves $f = 300$ MHz, $\frac{f}{\Delta f} = 50$ channels

light ($\lambda = 1\mu m$) $f = 3\times10^8$ MHz, $\Rightarrow 50\times10^6$ channels!

acceptance angle: $\sin\phi_c = \frac{n_c}{n_f} = \sin\left(\frac{\pi}{2} - \theta_t\right) = \cos\theta_t$

$$= \left(1 - \sin^2\theta_t\right)^{1/2}$$

$\theta_i < \theta_{max}$ with $n_i \sin\theta_{max} = n_f \sin\theta_t$

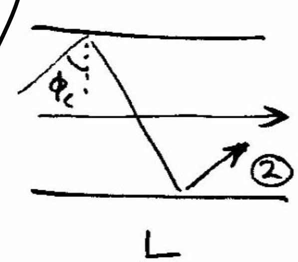
$$\Rightarrow \underline{\underline{\sin\theta_{max} = \frac{1}{n_i}\left(n_f^2 - n_c^2\right)^{1/2}}}$$

numerical aperture

$$NA \equiv n_i \sin\theta_{max} = \left(n_f^2 - n_c^2\right)^{1/2}$$

high NA $\Rightarrow$ will accept light from many angles.

travel time:



ray ①:

$$t_{min} = \frac{L}{v_f} = \frac{L}{c/n_f} = \frac{L n_f}{c}$$

ray ②:

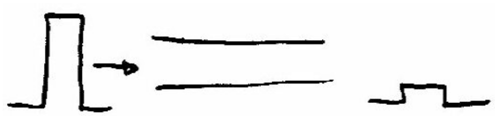
$$t_{max} = \frac{L/\sin\phi_c}{v_f} = \frac{L n_f/n_c}{c/n_f} = \frac{L n_f^2}{c\, n_c}$$

$$\frac{\Delta t}{L} = \frac{n_f}{c}\left(\frac{n_f}{n_c} - 1\right)$$

"modal dispersion"

- even for single mode (angle), n_f depends on λ (material dispersion)

Attenuation:



- bends
- evanescent waves
- absorption

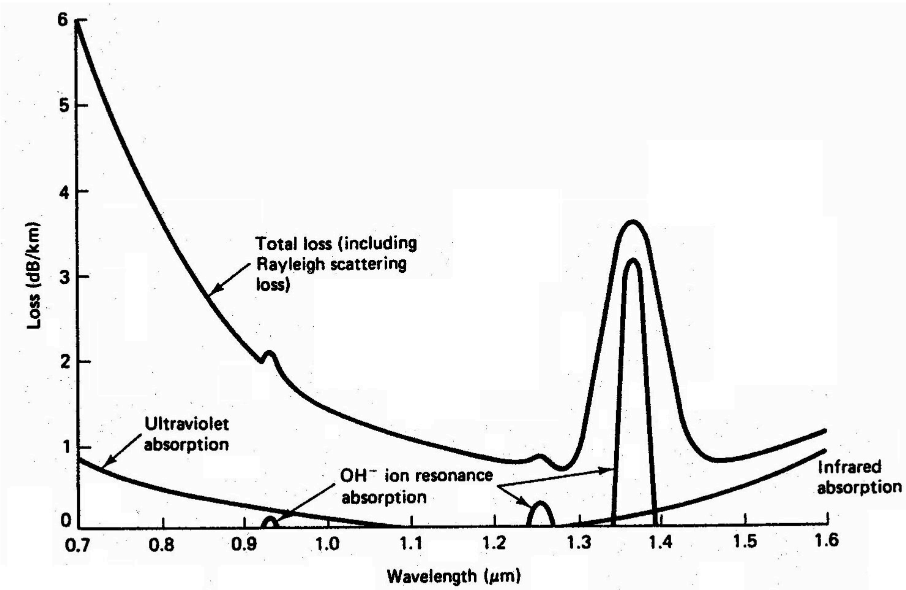


FIGURE 14–18 Absorption losses in optical fibers.

• minimum loss < 1 dB/km, at 1.55 μm

Material dispersion: $\propto d^2n/d\lambda^2$

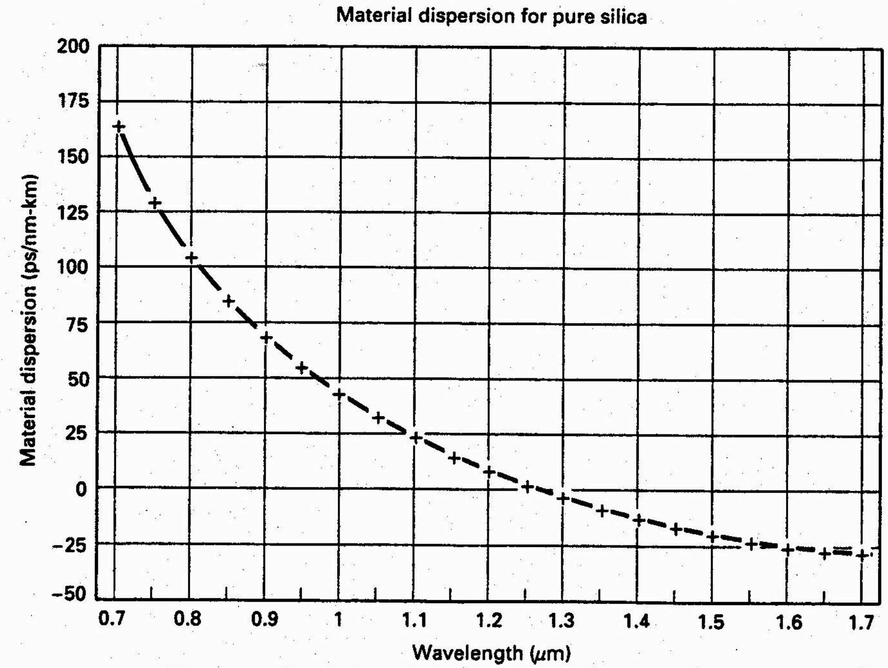


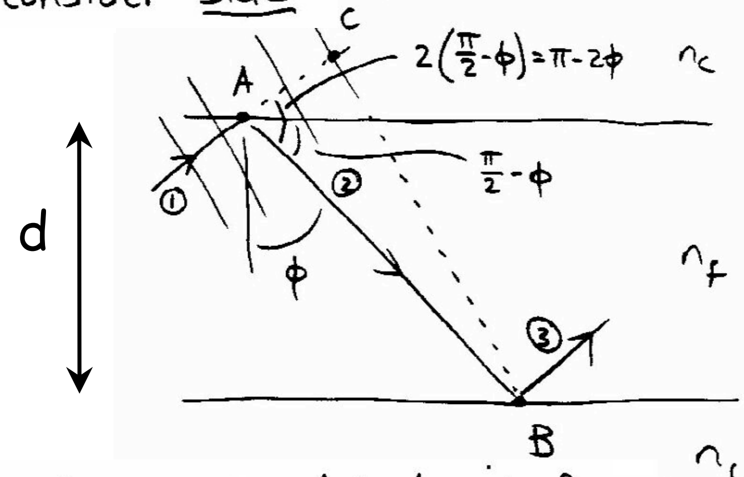
Figure 24-13 Material dispersion in pure silica. The quantity *M*, representing the pulse broadening (ps) per unit of spectral width (nm) per unit of fiber length (km), is plotted against the wavelength. Pulse broadening becomes zero at 1.27 μm and is negative as wavelength increases further.

• find minimum dispersion at 1.3 μm

Modes (see Pedrotti + Pedrotti, p. 501 ff)

- propogating wave must interfere constructively with itself:

- consider slab -



- want waves ① and ③ to interfere constructively at B:

$$\Rightarrow \quad OPL_{AB} - OPL_{AC} = \text{integral no. wavelength} \; ?$$

- also have phase shift due to reflections

$$\Rightarrow \frac{2\pi}{\lambda}\left(OPL_{AB} - OPL_{AC}\right) + 2\phi_r = 2\pi m$$

integer m

$$OPL_{AB} - OPL_{AC} = n_f \frac{d}{\cos\phi}\left(1 - \underbrace{\cos(\pi - 2\phi)}_{-\cos(2\phi)}\right)$$

$$= 2 n_f d \cos\phi$$

$$1 + \cos 2\phi = 2\cos^2\phi$$

$$\Rightarrow \quad 2\pi m = \frac{2\pi}{\lambda} 2 n_f d \cos\phi + 2\phi_r$$

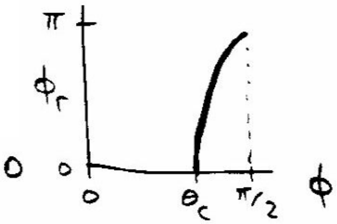
$$m = \frac{2 n_f d \cos\phi_m}{\lambda} + \frac{\phi_r}{\pi}$$

or

$$\boxed{m' = m - 1 = \frac{2 n_f d \cos\phi_{m'}}{\lambda} + \frac{(\phi_r - \pi)}{\pi}}$$

gives $\phi_{m'}$ for each m'

look at boundary conditions:



$$m' = 1 = \frac{2 n_f d \cos\phi_1}{\lambda} + \frac{\phi_{r_1} - \pi}{\pi} \approx 0$$

$$2 = \frac{2 n_f d \cos\phi_2}{\lambda} + \frac{\phi_{r_2} - \pi}{\pi}$$

$\vdots$

$$M = \frac{2 n_f d \cos\phi_M}{\lambda} + \frac{\phi_{r_M} - \pi}{\pi}$$

$\phi_M \approx \phi_c$

$= -1$

thus, number of modes,

$$M = \left\lfloor \frac{2n_f d \cos\phi_c}{\lambda} \right\rfloor - 1$$

"floor" (truncate)

$$= \left\lfloor \frac{2d}{\lambda} NA \right\rfloor - 1$$

this is only for slabs; for cylindrical geometry:

$$M = \frac{1}{2}\left(\frac{\pi d}{\lambda} NA\right)^2$$

eg/ $n_f = 1.60$, $n_c = 1.50$, $\phi_c = \sin^{-1}\left(\frac{1.5}{1.6}\right) = 70^\circ$

say, $d = 50\mu m$, $\lambda = 1\mu m$

$$\Rightarrow M = \left\lfloor \frac{2 \times 1.6 \times 50\mu m \times \cos 70^\circ}{1\mu m} \right\rfloor - 1 = 54$$

multimode fiber

for a single mode:

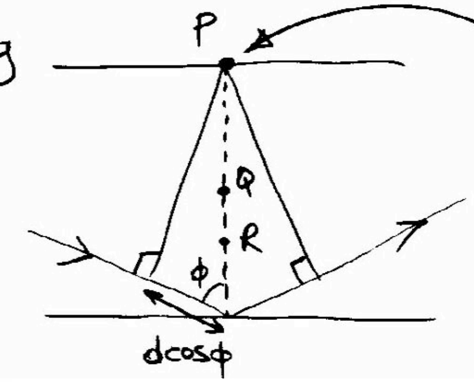
$$d < \frac{2\lambda}{2n_f \cos\phi_c} = \frac{1}{1.6 \cos\phi_c} = 1.8\,\mu m \ !$$

single mode

(usually n_f, n_c closer)

E-field profile in slab:

eg



phase diff at P:

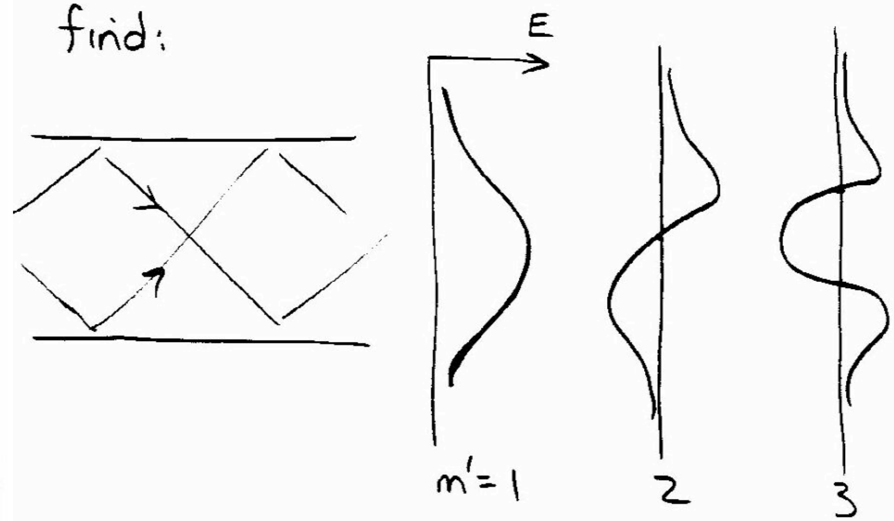
$$= 2n_f d \cos\phi \frac{2\pi}{\lambda} + \phi_R$$

$$= 2\pi m - \phi_R$$

destructive for $\phi_R \approx \pi$

look at intensity at other points Q, R, ...

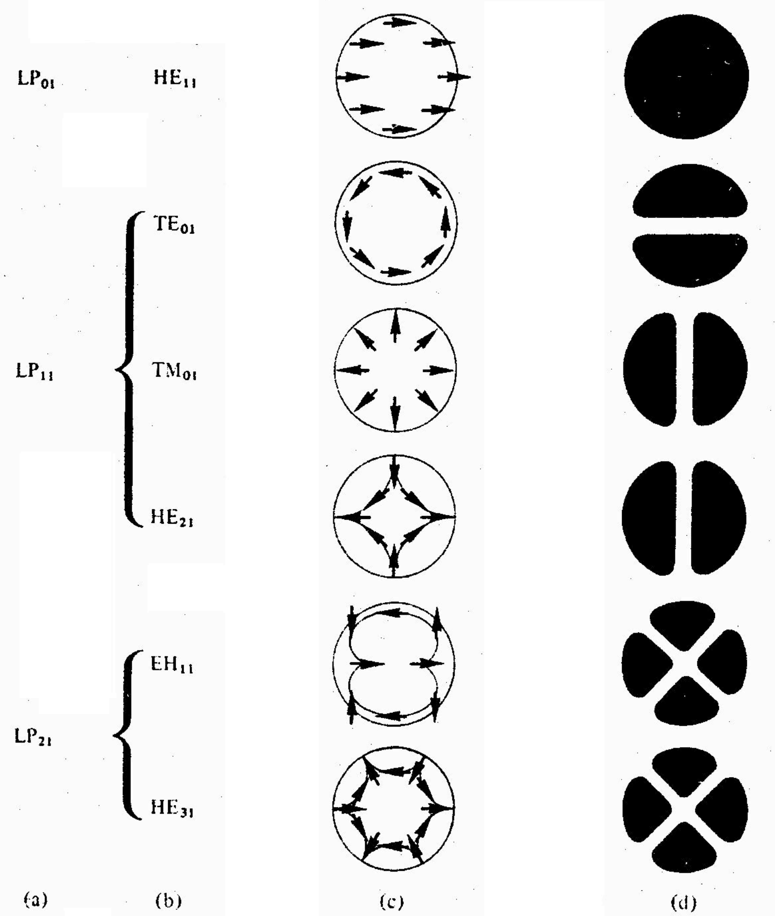
find:



note: lowest order mode is not "straight thru" (can't satisfy bc's).

Modes in an actual fiber:

OPTICAL FIBER COMMUNICATIONS: PRINCIPLES AND PRACTICE



The electric field configurations for the three lowest LP modes illustrated in terms of their constituent exact modes: (a) LP mode designations; (b) exact mode designations; (c) electric field distribution of the exact modes; (d) intensity distribution of E_x for the exact modes indicating the electric field intensity profile for the corresponding LP modes.